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EFFECT OF CROSS-TIE ON PERIOD OF C.C.M.

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1. Summary of Results

The device under consideration is a tie from the left side of the 1-wheel to the 4-wheel, so that the latter will step if there is an indent either on the right side of the 3-wheel (as usual) or on the left side of the 1-wheel. Its effect is considered with relation to the following three facts concerning the C.C.M. when all wheels have an even number of indents.

- (1) The period of the middle three wheels is 338.
- (2) The period of the whole machine is normally 4394.
- (3) When "defective" wheels are used, the period of the whole machine may be only 338.

Thus the setting AAAAA will become say KAAAS after 338 steps, the two end wheels precessing a constant amount, always even (here 10 and 18 respectively), and thus returning to their initial positions after  $13 \times 338 = 4394$  steps.

The cross-tie will evidently have no effect upon the 1-, 2-, and 3-wheels. We consider only its effect upon the 4-wheel. The behavior of the 5-wheel would require much deeper analysis.

If the cross-tie is used with normal wheels, it is found that the 4-wheel can never have a period of 338. It does, however, have what we might call a "statistical period" in the sense that there is a certain probability, ranging up to about one chance in fifteen, that it will have the same setting in any two particular machine positions 338 steps apart. Since this probability is so low, we are justified in saying that the cross-tie effectively breaks up the period of the middle three wheels. The same is true if the right side of the 3-wheel is defective, and consequently any short cycle on the 3-, 4-, and 5-wheels is destroyed.

The effect is somewhat different if the short cycle occurs on the 1-, 2-, and 3-wheels. In this case, for a given wheel order, certain initial settings may produce a short cycle even with the cross-tie in operation. The  $26^3$  settings of these three wheels fall into 52 distinct cycles of length 338. One expects three or four of these to be "bad", producing a cycle of 338 on the 4-wheel, and hence on all except possibly the 5-wheel. In a "good" cycle, the 4-wheel can never return to its initial setting after 338 steps.

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Using the cross-tie with normal wheels, the 4-wheel precesses a certain even amount  $d$  after 4394 steps. This amount is zero if the left side of the 1-wheel has the even-odd defect, in which case there exists a period of 4394 on all except possibly the 5-wheel. If, however, the left side of the 1-wheel is normal, the question of whether or not  $d = 0$  depends wholly on the 2- and 3-wheels. It would be practicable to find it out experimentally for the  $20 \times 18 = 360$  possible choices for these two wheels, in a set of ten wheels, but no simple criterion is apparent. The proportion of "bad" choices appears to be the random 1 in 13. When  $d = 0$ , the period of the first four wheels is  $13 \times 4394 = 57122$ .

When defective wheels are used, and a short cycle exists on the 1-, 2-, and 3-wheels, then in a "good" cycle the 4-wheel precesses a certain amount  $d$ , not zero and possibly odd, every 338 steps. Consequently the first four wheels will have a cycle of 4394 or 8788 or 676, depending on whether  $d$  is even, odd and not 13, or 13. The number of these various cases, including the number of "bad" cycles for which  $d = 0$ , is easily found for a given wheel order. With say, four defective wheels in a set of ten, they could be tabulated for the  $4 \times 12 \times 10 = 480$  possible choices of the 1-, 2-, and 3-wheels. In the examples, fictitious indent patterns are used.

For convenience, we assume throughout that when a wheel steps forward, the letters indicating its setting increase alphabetically.

## 2. Effect of Cross-ties at Intervals of 338.

Without the cross-tie, the 4-wheel returns to its initial setting after 338 steps. With the cross-tie, the 4-wheel will receive a certain number of "extra boosts" from the 1-wheel when the right side of the 3-wheel is in an inactive position. To find the displacement of the 4-wheel after 338 steps, we need only count the number of these "extra boosts" that it has received.

In Figure 1, assuming the indent patterns for the left side of the 2- and 3-wheels as shown, 338 successive settings of the 1-, 2-, and 3-wheels are given, starting at AAA. Those of the 3-wheel are the normal alphabet, repeated for each column, and are omitted. The 339th position would be YAA, and the next 338 settings differ from those shown in Figure 1 only in that the 1-wheel is two steps behind (Y instead of A, Z instead of B, A instead of C, etc., throughout).

In Figure 2, indent patterns are assumed for the left side of the 1-wheel and the right side of the 3-wheel. The 3-wheel is in the inactive setting E just 13 times during this block of 338 steps, and reference to Figure 1 shows that the 1-wheel

is at these times in the settings

B O Z L C K U C Q I T D O

Of these, the settings D, L, O, Q, and T (underlined) are active positions of the 1-wheel, and the rest are inactive. Consequently the 4-wheel receives six extra boosts while the 3-wheel is in setting E. By adding up the boosts for all the inactive positions (E, F, I, J, N, Q, S, T, X, and Z) of the 3-wheel, we find the desired displacement of the 4-wheel after the 338 steps shown in Figure 1. For this purpose a frequency count is made of the  $10 \times 13 = 130$  letters (settings of the 1-wheel) in these rows of Figure 1. The result given in Figure 2. One extracts from this count the active settings of the 1-wheel:

|   |           |
|---|-----------|
| A | 4         |
| D | 4         |
| G | 4         |
| H | 2         |
| L | 4         |
| O | 10        |
| P | 3         |
| Q | 3         |
| T | 7         |
| X | 3         |
|   | <u>44</u> |

Since the total is 44, the 4-wheel has received 44 extra boosts in these 338 steps. If it starts at A it will end at  $A + 44 = A + 18 = S$ .

If we started at position ZAA-- instead of AAA-- in Figure 1, every letter giving a setting of the 1-wheel is diminished by one alphabetically. If we made a new frequency count for the rows in which the 3-wheel is inactive, we would find as many A's as formerly there were B's, as many D's as formerly there were E's, etc. Hence we can find the number of extra boosts as the sum of the original frequencies of B, D, H, I, etc. These are entered in Figure 2 in the column headed Z, and the total is 41.

Since, in the example, the 1-wheel moves from A to Y in 338 steps, then to W, etc., the position of the 4-wheel is found from the upper line of totals (44, 56, 44, 49, etc.) in Figure 2. The successive positions of the 4-wheel after each batch of 338 are therefore as follows:

|        |   |        |   |   |
|--------|---|--------|---|---|
| A + 44 | = | A + 18 | = | S |
| S + 56 | = | S + 4  | = | W |
| W + 44 | = | W - 8  | = | O |
| O + 49 | = | O - 3  | = | L |
| L + 55 | = | L + 3  | = | O |

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O + 39 = O - 13 = B  
 B + 55 = B + 3 = E  
 E + 52 = E + 0 = E  
 E + 51 = E - 1 = D  
 D + 40 = D + 14 = R  
 R + 54 = R + 2 = T  
 T + 52 = T + 0 = T  
 T + 49 = T - 3 = Q

Twice in the above 13 times the 4-wheel returned to its starting point (E and T respectively) but there is no evidence of a period. Figure 3 indicates the absence of a period a little better. Here the settings of all but the 5-wheel are given for the first 31 positions of successive batches of 338 steps. Active positions of the left side of the 1-wheel and the right side of the 3-wheel are indicated by arrows. Of the 62 comparisons made here, between a given position and that 338 steps later, just 4 have the middle three wheels in the same position (IUK, IVL, JWM, and JKN). These are all in a lump, comprising "Zone 8". A zone is a stretch of active positions of the 3-wheel. Within a zone, the displacement of the 4-wheel is the same. If one begins at CDF-- instead of AAA-- in Figure 1, the frequency count is altered only in that one Z is added and one B is removed. Beginning anywhere in Zone 2 (CDG, CDH, or CEI), one adds an A and removes a C, in addition to the above Z - B exchange. Proceeding thus from zone to zone, we get the frequency changes noted in Figure 2. Increases or decreases in the resulting totals are shown by +'s and -'s. Places where the total equals 52 are encircled. Considering the width of the zones, there are 44 such repetitions in 676 comparisons, as seen in the following tabulation:

| <u>ZONE</u> | <u>WIDTH</u> | <u>NO. 52's</u> | <u>TOTAL 52's</u> |
|-------------|--------------|-----------------|-------------------|
| 0           | 5            | 3               | 15                |
| 1           | 1            | 1               | 1                 |
| 2           | 3            | 3               | 9                 |
| 3           | 1            | 0               | 0                 |
| 4           | 4            | 0               | 0                 |
| 5           | 3            | 0               | 0                 |
| 6           | 2            | 0               | 0                 |
| 7           | 1            | 1               | 1                 |
| 8           | 4            | 3               | 12                |
| 9           | 2            | 3               | 6                 |
|             |              |                 | <u>44</u>         |

From the manner in which the totals change it is clear that a true period of 338 could exist for the 4-wheel only in the absurdly trivial case in which the 1-wheel had an indent in every other position. The example chosen indicates 44 chances out of 676, or 1 chance in 15.4, of the 4-wheel being in the same setting at two positions 338 steps apart. Considering the selection of 10 letters of the frequency count as a random matter, the probability that the total thereof is 52 is

$${}_{130}C_{52} \left(\frac{10}{26}\right)^{52} \left(\frac{16}{26}\right)^{78} = \frac{1}{14.95}$$

or very close to 1 in 15. Here the mean sum is  $5 \times 10 = 50$ . With 12 inactive places on the 3-wheel, it would be  $5 \times 12 = 60$ , and the probability of hitting 52 (or 78) considerably less.

### 3. Effect of Cross-tie at Intervals of 4394.

To see what happens to the 4-wheel after the normal cycle of 4394 steps when the cross-tie is introduced, we observe that it will be advanced by an amount equal to the sum of every other column of Figure 2 ( $44 + 56 + 44 + 49 + \dots$  or  $41 + 67 + 44 + 44 + \dots$ ). In general terms, let  $k$  be the number of notches on the right side of the 3-wheel, so that  $26 - k$  is the number of inactive places. Then the total of the frequency count (130 in Figure 2) is  $13(26 - k) = N$ , and  $N$  is divisible by 26 since  $k$  is even for the type of wheels we are considering. Let  $N_e$  and  $N_o$  be the totals of the even and odd terms; in the example

$$\begin{aligned} N_e &= 7 + 4 + 2 + \dots = 60 \\ N_o &= 4 + 11 + 3 + \dots = 70 \end{aligned}$$

Let  $a$  be the number of indents on the left side of the 1-wheel in the even positions, and  $b$  the number in the odd positions. In the example

$$\begin{aligned} a &= 6 \quad (D, H, L, P, T, X) \\ b &= 4 \quad (A, G, O, Q) \end{aligned}$$

Then the desired total is

$$aN_e + bN_o = (a-b)N_e + bN \equiv (a-b)N_e \pmod{26}$$

In the example,  $(a-b)N_e = 2 \times 60 = 120 \equiv 16 \pmod{26}$ . Thus A will change to  $A + 16 = Q$  after 4394 steps, as already noted above. If one started at BAAA- instead, one would get the negative (mod.26) of this.

If the left side of the 1-wheel has the odd-even defect, namely  $a = b$ , then the 4-wheel will always return to its initial position after 4394 steps. In the contrary case, it depends on whether or not  $N_e$  is divisible by 26, or actually by 13, since  $a - b$  is even. This number  $N_e$  depends only on the indent patterns of the wheels, and not on the initial setting. For the 1-, 2-, and 3-wheels still have the period of 4394. Or, viewed otherwise, the successive changes in the frequency count consist always of adding one even letter and taking another away, or adding one odd letter and removing another odd letter.  $N_e$  could be found experimentally for each of the  $360 = 20 \times 18$  choices of 2- and 3-wheels, in a set of 10 wheels, but no



a priori way of avoiding its being divisible by 13 is apparent.

If  $N_e$  is not divisible by 13, then the 3-wheel evidently precesses the amount  $(a-b)N_e$  every 4394 steps, coming back to its original setting only after  $13 \times 4394$  steps.

#### 4. Effect of Cross-tie When Short Cycle Exists.

In Figure 4, one indent on the left face of the 2-wheel has been changed (Z to W) in order to produce a short cycle. In Figure 5, the appropriate frequency count and totals are shown. In this case, since the 1-wheel returns to its initial setting after 338 steps, there are no changes to be made in the frequency count as we progress along the same cycle. In other words, we find that the 4-wheel gets 47 boosts in the 338 steps starting with AAA--, so that AAAA- will become AAAV-, and we see that the 4-wheel also gets 47 boosts if we start at CDF--, or anywhere else in the same cycle.

The 263 initial settings of the 1-, 2-, and 3-wheels break up into 52 cycles of length 338. For each of these cycles, the 4-wheel precesses a constant amount  $d$  every 338 steps, and  $d$  may have any value at all from 0 to 25. For the cycle starting AAA,  $d = 47 = -5$ . In Figure 5 the totals give the values of  $d$  for each of the 26 cycles starting at AAA, BAA, ..., ZAA. The values of  $d$  for the other 26 cycles starting at ABA, BBA, ..., ZBA can be found similarly.

The expected proportion of "bad" cycles, with  $d = 0$ , seems no different than the theoretical  $1$  in  $15$  occurring above, but will of course fluctuate widely from wheel order to wheel order. In the example, not a single bad cycle exists, at least for half of the 52 cycles.

The period of the 4-wheel will thus be 338, 676, 4394, or 8788, depending on whether  $d$  is 0, 13, even, or prime to 26. Note the occurrence of  $d = 65 = 13$  for the cycle starting GAA in the example.

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FIG. 1 338 SUCCESSIVE POSITIONS OF 1- AND 2- WHEELS.

|   | 2L | 3L | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| A |    |    | AA | KK | VU | KE | ZO | JY | UI | BS | NC | FM | TW | CG | QQ |
| B | X  |    | AA | LK | WU | KE | AO | JY | UI | BS | OC | GM | TW | CG | QQ |
| C | X  | X  | AA | MK | XU | KE | BO | JY | UI | BS | PC | HM | TW | CG | QQ |
| D |    | X  | AB | NL | YV | KF | CP | JZ | UJ | BT | QD | IN | TX | CH | OR |
| E |    | X  | BC | OM | ZW | LG | CP | JZ | UJ | BT | QE | IO | TY | CH | OR |
| F | X  |    | CD | PN | ZX | LH | CR | KA | UK | CU | QF | JP | TZ | DI | OS |
| G |    |    | CD | PN | ZX | MH | CR | KB | VL | DV | RF | JP | UZ | DJ | OT |
| H | X  | X  | CD | PN | ZX | NH | CR | LB | WL | EV | SF | JP | VZ | DJ | PT |
| I |    | X  | CE | PO | ZY | DI | CS | NC | YM | FV | TG | JP | WA | DJ | QT |
| J |    | X  | CE | QO | ZY | OI | CS | OC | ZM | GW | TG | JQ | WA | DJ | RU |
| K | X  |    | CF | RP | ZZ | OJ | CT | PD | AN | GX | TH | JR | WB | EK | SU |
| L | X  |    | DF | RP | AZ | OJ | DT | PD | AN | GX | UH | JR | WB | FL | TV |
| M | X  | X  | EF | RP | BZ | OJ | ET | PD | AN | GX | VH | JR | YB | GL | UV |
| N | X  | X  | FG | RQ | CA | OK | FU | PE | AO | GY | WI | JS | ZC | IM | VV |
| O |    | X  | FH | RR | CB | PL | GV | PF | BP | GZ | WJ | JS | AD | IM | WW |
| P | X  |    | GH | RR | DB | QL | HV | QF | BP | HZ | WJ | KT | AD | JN | WX |
| Q |    | X  | HH | RR | EB | RL | IV | RF | BP | IZ | WJ | LT | AD | JN | WX |
| R |    |    | II | RS | FC | SM | JW | SG | BQ | JA | WK | MU | AE | JO | WY |
| S |    |    | II | RS | GC | TM | JW | SG | BQ | JA | XK | NU | AE | KO | WY |
| T | X  |    | II | RS | HC | UM | JW | SG | BQ | JA | YK | OU | AE | LO | WY |
| U | X  | X  | II | RS | IC | VM | JW | SG | BQ | JA | ZK | PU | AE | MO | WY |
| V | X  | X  | II | RS | JC | WM | JW | SG | BQ | JA | AK | QU | AE | NO | WY |
| W |    | X  | IJ | RT | KD | XN | JX | SH | BR | JB | BL | RV | AF | OP | WZ |
| X |    | X  | IJ | ST | KD | XN | JX | TH | BR | KB | CL | SV | BF | OP | XZ |
| Y |    |    | IK | TU | KE | XO | JY | UI | BS | LC | DM | TW | CG | QQ | YA |
| Z | X  |    | JK | UU | KE | YO | JY | UI | BS | MC | EM | TW | CG | QQ | YA |

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FIG. 2 FREQUENCY & CHANGES

|   | <u>1L</u> | <u>3R</u> | ADD:<br>SUB: | Z<br>B | A<br>C | A<br>C | A<br>C | D<br>F | F<br>H | G<br>I | G<br>I | G<br>I | H<br>J |
|---|-----------|-----------|--------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| A | X         | X         | 4            | 4      | 5      | 6      | 7      | .....  | .....  | .....  | .....  | .....  | 7      |
| B |           | X         | 7            | 6      | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 6      |
| C |           | X         | 11           | 11     | 10     | 9      | 8      | .....  | .....  | .....  | .....  | .....  | 8      |
| D | X         | X         | 4            | .....  | .....  | 4      | 5      | .....  | .....  | .....  | .....  | .....  | 5      |
| E |           |           | 3            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 3      |
| F |           |           | 2            | .....  | .....  | .....  | 2      | 1      | 2      | .....  | .....  | .....  | 2      |
| G | X         | X         | 4            | .....  | .....  | .....  | .....  | 4      | 5      | 6      | 7      | .....  | 7      |
| H | X         | X         | 2            | .....  | .....  | .....  | .....  | 2      | 1      | .....  | .....  | 1      | 2      |
| I |           |           | 7            | .....  | .....  | .....  | .....  | 7      | 6      | 5      | 4      | .....  | 4      |
| J |           |           | 12           | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 12     | 11     |
| K |           | X         | 6            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 6      |
| L | X         | X         | 4            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 4      |
| M |           | X         | 1            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 1      |
| N |           |           | 2            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 2      |
| O | X         | X         | 10           | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 11     |
| P | X         | X         | 3            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 3      |
| Q | X         |           | 3            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 3      |
| R |           | X         | 7            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 7      |
| S |           |           | 5            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 5      |
| T | X         |           | 7            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 7      |
| U |           | X         | 4            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 4      |
| V |           | X         | 1            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 1      |
| W |           | X         | 8            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 8      |
| X | X         |           | 3            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 3      |
| Y |           | X         | 4            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 4      |
| Z |           | X         | 6            | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 6      |
|   |           |           | 130          | 7      | .....  | .....  | .....  | .....  | .....  | .....  | .....  | .....  | 7      |

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FIG. 4

|   | <u>2L</u> | <u>3L</u> | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |    |
|---|-----------|-----------|----|----|----|----|----|----|----|----|----|----|----|----|----|
| A |           |           | AA | KK | VU | IE | XO | MY | WI | DS | OC | GM | WW | FG | RQ |
| B | X         |           | AA | LK | WU | IE | YO | MY | WI | DS | PC | HM | XW | FG | RQ |
| C | X         | X         | AA | MK | XU | IE | ZO | MY | WI | DS | QC | IM | YW | FG | RQ |
| D |           | X         | AB | NL | YV | IF | AP | MZ | WJ | DT | RD | JN | ZX | FH | RR |
| E | X         |           | BC | OM | ZW | JG | AQ | MA | WK | EU | RE | JO | ZY | GI | RS |
| F |           |           | CD | PN | AX | JH | AR | MA | XL | FV | RF | KP | ZZ | GJ | RT |
| G | X         | X         | CD | PN | AX | KH | AR | NB | YL | GV | SF | KP | ZZ | GJ | ST |
| H |           |           | CD | PN | AX | LH | AR | OB | ZL | HV | TF | KP | ZZ | GJ | TT |
| I | X         | X         | CE | PO | AY | MI | AS | PC | AM | IW | UG | KQ | ZA | GK | UU |
| J |           |           | CE | QO | AY | MI | AS | QC | BM | JW | UH | KQ | ZA | HK | VU |
| K | X         | X         | CF | RP | AZ | MJ | AT | RD | CN | KX | VH | KR | ZB | IL | WV |
| L | X         |           | DF | RP | AZ | MJ | BT | RD | CN | KX | WH | KR | AB | JL | XV |
| M | X         | X         | EF | RP | AZ | MJ | CT | RD | CN | KX | XI | KR | BB | KL | YV |
| N |           |           | FG | RR | AA | MK | DU | RE | CO | KY | XJ | KS | CC | LM | ZW |
| O | X         |           | FH | RR | AB | NL | EV | RF | DP | KZ | XJ | KT | DD | MN | AX |
| P |           | X         | GH | RR | BB | OL | FV | SF | DP | KZ | XJ | LT | DD | MN | AX |
| Q |           |           | HH | RR | CB | PL | GV | TF | DP | KZ | XJ | MT | DD | MN | AX |
| R |           |           | II | RS | DC | QM | HW | UG | DQ | KA | XK | NU | DE | MO | AY |
| S |           |           | II | RS | EC | RM | IW | UG | DQ | KA | YK | OU | DE | NO | AY |
| T | X         |           | II | RS | FC | SM | JW | UG | DQ | KA | ZK | PU | DE | OO | AY |
| U | X         |           | II | RS | GC | TM | KW | UG | DQ | KA | AK | QU | DE | PO | AY |
| V | X         | X         | IJ | RT | HC | UM | LW | UH | DR | KB | CL | RU | DE | QO | AY |
| W |           |           | IJ | ST | ID | VN | MX | VH | DR | LB | DL | SV | DF | RP | AZ |
| X |           | X         | IK | TU | IE | VN | MX | WI | DS | MC | EM | TV | FG | RP | AZ |
| Y |           |           | JK | UU | IE | VO | MY | WI | DS | NC | FM | UW | FG | RQ | AA |
| Z |           |           |    |    |    | WO |    |    |    |    |    | VW |    | AA | AA |

FIG. 5

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|   |    |   |   |   |    |    |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |     |
|---|----|---|---|---|----|----|---|---|---|---|---|---|---|---|---|---|---|---|----|---|---|---|---|---|---|---|---|-----|
| A | 14 | 2 | 2 | 6 | 10 | 3  | 5 | 4 | 2 | 7 | 6 | 8 | 2 | 9 | 2 | 3 | 5 | 2 | 12 | 2 | 2 | 6 | 4 | 3 | 3 | 1 | 7 | 130 |
| B | 2  | 3 | 5 | 6 | 4  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2   |
| C | 6  | 7 | 7 | 6 | 8  | 8  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| D | 10 | 2 | 5 | 4 | 6  | 2  | 5 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| E | 3  | 3 | 7 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| F | 5  | 7 | 6 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| G | 4  | 9 | 4 | 2 | 3  | 12 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| H | 2  | 7 | 7 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| I | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| J | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| K | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| L | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| M | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| N | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| O | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| P | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| Q | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| R | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| S | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| T | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| U | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| V | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| W | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| X | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| Y | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| Z | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |
| A | 2  | 2 | 2 | 2 | 2  | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |     |

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