 17 April 1944

TOP SECRET

THE PERIOD OF THE CCM

The CCM has five wheels of which the center steps every time, and the others irregularly.

Designating the wheels by 1,2,3,4,5, in order, the 3 wheel steps continually, the 2 and 4 wheels stepping is under control of the 3 wheel, and the 1 and 5 wheels step under the the influence of the 2 and 4 wheels respectively.

Since the stepping of the 4 and 5 wheels is similar (differing only in the notches) to the stepping of the 2 and 1 wheels, we need only discuss the motion of the 3-4-5 system.

Let there be k notches on the 3 wheel which induce the motion of the 4 wheel.

CASE I. k is relatively prime to 26.
After the 3 wheel has revolved 26 times, each of the k notches has stepped the 4 wheel 26 times. Hence the 4 wheel has stepped 26k times or k complete revolutions.

When the 4 wheel comes to the A position under the influence of (say) the 1st notch of the 3 wheel it stays until the 2nd notch of the 3 wheel moves it on to B position. The time elapsed in the A position is the interval on the 3 wheel between the first and second notches. During the cycle the A position is attained under the action of <u>each</u> of the k notches, and the total time elapsed in the A position is the sum of the intervals on the 3 wheel between the first and second notches, the second and third notches, the (k-1)st and kth notches and the kth and 1st notches. This sum is clearly once around the wheel or 26.

By a similar argument each position of the 4 wheel is up 26 times during the cycle. Hence the 5 wheel has been kicked 26 times by each notch of the 4 wheel and hence has turned an integral number of times.

This shows that under the hypothesis k-relatively prime to 26, (where k is the number of notches on the center wheel) the period is 676.

CASE II. k is an even number, 2m.
After 13 revolutions of the 3 wheel, the 4 wheel has advanced 13 x 2m = 26m positions or m complete revolutions.

ENCLOSURE (B)

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THE PERIOD OF THE CCM (Continued)

As before, if the A position of the 4 wheel is caused by the 1st notch of the 3 wheel, the A position remains for an interval equal to the interval between 1st and 2nd notches of the 3 wheel. During the 13 revolutions of the 3 wheel, the A position of the 4 wheel comes up m times under the influence (in some order) of the 1st, 3rd, 5th, (2m-1)st notches of the 3 wheel and the total time elapsed in the A position is the sum of the intervals on the 3 wheel between the 1st and 2nd notches, the 3rd and 4th notches, the 5th and 6th notches, ..., and the (2m-1)st and 2m th notches. Let this sum be q. The B position of the 4 wheel comes up from the 2nd, 4th, 2m th notches of the 3 wheel, and the total elapsed times the sum of the interval between the 2nd and 3rd notches, the 4th and 5th, ..., and the 2m th and 1st notches. This sum is 26-q. The C position comes up from notches 1, 3, 5, ... on the 3 wheel, and the total elapsed time is q. Thus the positions A,C,E, ..., Y have elapsed time q, and the positions B,D,F, ..., Z have elapsed time 26-q.

The total amount of kick received by the 5 wheel is

$$qr_1 + (26-q)r_2$$

where r_1 is the number of notches of the 4 wheel in positions A,C,E, ..., Y, and r_2 is the number of notches of the 4 wheel in positions B,D, ... Z.

The cycle of the whole 3-4-5 system depends on the factors common to $qr_1 + (26-q)\ r_2$ and 26.

The cycle is 338 if

$$qr_1 + (26-q)\ r_2 - o \pmod{26}$$

i.e.
$$q(r_1-r_2) = o \pmod{26}$$

Thus the cycle is short if

a) $r_1 = r_2$

b) $q = 13$ and $r_1 - r_2 = o \pmod 2$

Since all wheels have an even number of notches to prevent a short cycle occurring as in Case I, condition b) is simply $q = 13$.

Several wheels in the present set have been examined and several satisfy condition a). When such a wheel is in position 4 the cycle of the 3-4-5 system is 338. Similarly when such a wheel is in position 2 the cycle of the 3-2-1 system is 338 and the cycle of the whole machine is short.

If no wheels satisfy either a) or b) on either rim, then the minimum period would be 4394 letters.